# ASSIGNMENT SET – I

## Mathematics: Semester-II

## M.Sc (CBCS)

## Department of Mathematics

## Mugberia Gangadhar Mahavidyalaya

**PAPER - MTM-202**

**Paper: Numerical Anaysis**

**Answer all the questions**

1. Develop the cubic spline of the following information

| | | | | |
|---|---|---|---|---|
| x: | 1 | 2 | 3 | 4 |
| f(x): | 1 | 5 | 11 | 8 |

where $y''(1) = 0 = y''(4)$. Hence compute $y(1.5)$.

2. Approximate the function $\sin x, -1 \le x \le 1$ using Chebyshev polynomials.
3. What are the basic differences between interpolation and approximation?
4. Find whether the following function is spline or not? $f(x) = \begin{cases} -x^2 - 2x^3, & x \in [-1, 0] \\ -x^2 + 2x^3, & x \in [0, 1] \end{cases}$
5. Express the polynomial $x^4 + 2x^3 - x^2 + 5x - 9$ in terms of Chebyshev polynomials.
6. Define cubic spline.
7. Is it possible to write any polynomial in terms of the Tchebyshev polynomial? Explain with an example.
8. Use the 3-point Gauss-Tchebyshev quadrature method to find the value of
$$\int_{-1}^{1} (\sec x + 2x^4)\, dx.$$
9. State the minimax principle of polynomial interpolation.
10. Write down the sufficient conditions for the convergence of fixed point iteration method.

11. What are the advantages to approximate a function using orthogonal polynomials?
12. Define spline interpolation.
13. Compare Lagrange's interpolation and cubic spline interpolation methods.
14. Let $f(x) = \begin{cases} 2x3 - 4.5x2 + 3x + 5, & 0 \le x \le 1 \\ -x3 + 4.5x2 - 6x + 8, & 1 \le x \le 2. \end{cases}$

    Show that *f(x)* is a cubic spline.
15. Describe approximation of a continuous function using orthogonal polynomials.
16. Use Tchebyshev polynomials to find least squares approximation of second degree for $f(x) = (1-x^2)5/2$ on the interval [-1,1].
17. What is the residual?
18. Let (2,2), (-1.5, -2), (4,4.5) and (-2.5,-3) be a sample. Use least squares method to fit the line y = a+bx based on this sample and estimate the total error.
19. Derive the periodic spline interpolation of a continuous function y=f(x) in [a, b].
20. Use least square method to solve the following system of equations: $x + 3y = 1,\ x - y = 5,\ -3x + y = 4,\ 3x + 2y = 7$
21. Describe cubic-spline interpolation method in case of natural spine.
22. Find the value of b when $\varphi(x) = \begin{cases} p0(x), & 0 \le x \le 1 \\ p1(x), & 1 \le x \le 2 \end{cases}$

    Where, $p_0(x) = 0.98x^3 - 0.68x^2 + 0.2x,\ 0 \le x \le 1$

$P_1(x) = -1.04(x-1)^3 + 2.26(x-1)^2 + 1.78(x-1) + b,\ 1 \le x \le 2$

    is a cubic spline.
23. Find a Cubic spline curve that passed through (0, 0.0), (1, 0.5), (2, 2.0), (3, 1.5) with natural and boundary condition y"(0)=y"(3)=0.
24. Use Chebyshev polynomial find least square approximation of second degree for $f(x) = \sqrt{1-x^2}$ in [-1, 1].
25. Derive the Gauss-Chebyshev quadrature formula. Using six points Gauss-Chebyshev quadrature formula evaluate $\int_0^2 \frac{x}{1+x^3}dx$.
26. What are the basic differences between interpolation and appromimation?
27. Discuss the Newton-Raphson method for a pair of non-linear equations with stated convergence conditions and convergences rate.
28. Explain Monte Carlo method to integrate $\int_a^b f(x)dx$.
29. Derive the Gauss-Legendre quadrature formula to integrate $\int_{-1}^1 \psi(x)f(x)dx$

30. State the sufficient condition for convergence of the Gauss-Seidal iteration method to solve a system of non-linear equations containing three equations and three variables.
31. What are the advantages of Gaussian quadrature compared to Newton-Cote quadrature?
32. What do you mean by Newton-Cotes open type quadrature formulae? Explain.
33. Describe 3-point Gauss – Legendre quadrature formula.
34. Deduce 3-point Gauss-Legendre quadrature formula. Using this formula find the value of

$$\int_0^2 \frac{\sin x}{1+x}\,dx$$

35. Find the inverse of the following matrix using partial pivoting

$$\begin{pmatrix} 0 & 2 & 4 \\ 1 & 2 & -5 \\ 4 & 2 & 6 \end{pmatrix}$$

36. Describe Braistow's method to find all roots of an algebraic equation of degree *n*.
37. Explain the Barstow method to find all roots of a polynomial equation.
38. Use Newton-Raphson method to solve the following system of non-linear equations'(x,y)=0,g(x,y)=0.

39. Solve the following problem:

$$\frac{\partial^2 u}{\partial t^2} = 16\frac{\partial^2 u}{\partial x^2}\,, t > 0, 0 < x < 1$$

The initial conditions are $u(x,0) = f_1(x)$and$\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = f_2(x),\ 0 < x < 1$ and the boundary conditions are $u(0,t) = g_1(t)$ and $u(1,t) = g_2(t), t \geq 0.10$

40. What do you mean by single step and multistep methods?
41. Discuss the Gauss-Seidel iteration method to solve a system of non-linear equations.
42. What are the advantages of the predictor-corrector method to solve an ODE?
43. Compare Lagrange's interpolation and cubic spline interpolation methods.
44. What do you mean by partial and complete pivoting methods to find the inverse of a matrix? What is the advantage to use such methods?
45. What do you mean by Newton-Cotes open type quadrature formulae? Explain.
46. Describe minimax polynomial.
47. What do you mean by single-step and multi-step methods to solve an ODE? Give examples of such methods. What is the main drawback of the multi-step method?
48. Discussed Newton-Rapson's method to solve the following non-linear equations

$$f(x,y) = 0, \qquad g(x,y) = 0.$$

49. Economize the power series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

correct up to four significant figures.

50. Deduce 1-point and 2-point Gauss-Legendre quadrature formulae.
51. Discuss the stability of the second-order Runge-Kutta method.
52. Let us consider the wave equation

$$u_{tt} = c^2 u_{xx}, \quad t > 0,\ 0 < x < 1$$

where initial conditions $u(x,0) = f(x)$ and $u_t = g(x), 0 < x < 1$ at $(x,0)$ and boundary conditions

$$u(0,t) = \phi(t) \text{and } u(1,t) = \psi(t), \qquad t \geq 0.$$

Describe a finite difference method to solve the above problem.

53. Describe the power method to find the largest eigenvalue and corresponding eigenvector of a matrix.
54. Describe the LU-decomposition method to solve a system of linear equations.
55. Suppose a table of values $(x_i, y_i), i = 0, 1, 2, \ldots, n$, is given. Describe the natural cubic spline method to fit this set of data.
56. Use Jacobi's method to determine all eigenvalues and the eigenvectors of the real symmetric matrix $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$.
57. Explain the ill-conditioned and well- conditioned system. The coefficient matrices of two system of equations are $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ *and* $B = \begin{pmatrix} 1 & 1 \\ 1 & 1.0001 \end{pmatrix}$. Find the condition numbers of two systems and indicate which system is stable.
58. Solve the system of equations

$$\begin{aligned} 2x + 4y - 2z &= 14 \\ x + 3y - 4z &= 16 \\ -x + 2y + 3z &= 1 \end{aligned}$$

using LU-decomposition method.

59. Solve the boundary value problem $y'' + xy' + 4 = 0, y(0) = 0$ *and* $y(1) = 0$ with step length $h = 0.25$.
60. Write the merits and demerits of the LU-decomposition method to solve a system of linear equations.
61. Suppose for a system of linear equations $AX = B$, the matrix $A$ is decomposed as $A = LU$, where $L$ and $U$ are the lower and upper triangular matrices and they are known. Explain a suitable method to solve the equation $AX = B$ with the help of the matrices $L$ and $U$.
62. Explain the successive overrelaxation method to solve a system of linear equations.
63. Explain the Gauss-Lengendre quadrature formula for *n* nodes.
64. Describe Jacobi's method to find all eigenvalues and eigenvectors of a symmetric matrix.
65. Find the value of y(0.20) for the initial value problem
    $\frac{dy}{dx}$ = xy +2y with y(0)=1 using Milne's predictor-corrector method, taking h=0.05

    Discuss fixed point iteration method to solve the following non-linear equations: f (x,y) = 0, g (x,y) = 0.

a) Find the largest eigenvalue in magnitude and the corresponding eigen vector of the matrix

$$\mathbf{A} = \begin{matrix} 2 & 3 & 1 \\ 4 & 3 & 4 \\ 7 & 6 & 1 \end{matrix}$$

66. Describe LU – decomposition method to solve a system of linear equations.
67. Using LU decomposition method solve the following system of equations: $2x + 3y - 6z = 2,\ 3x + y + 2z = 7,\ -3x + 2y + 6z = 7$
68. Describe relaxation method to solve a system of linear equations.
69. Explain Gauss-Jacobi Iterative method to solve a system of linear equations.
70. Solve the following system of linear equations by Gauss-Jacobi Iterative method. $x + y + 4z = 9,\quad 8x - 3y + 2z = 20, 4x + 11y - z = 33$
71. Explain the finite difference method to solve the following IVP.

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x), y(x_0) = a, y'(x_0) = b,$$

where $P(x), Q(x)$ and $R(x)$ are continuous on $[x_0, x_n]$

1. Solve the following system of linear equations by Gauss-Jacobi Iterative method.
   x+y+4z=9

   8x-3y+2z=20

   4x+11y-z=33

72. Use fourth order Runge-Kutta method to solve the second order initial value problem $2y''(x) - 6y'(x) + 2y(x) = 4e^x$ with $y(0) = 1$ and $y'(0) = 1$ at $x = 0.2, 0.4$.
73. Discuss Milne's predictor-corrector formula to find the solution of $y' = f(x\ y), y(x_0) = y_0$.
74. Analyze the stability of Euler's method for initial value ODE.
75. Describe a implicit method to solve a parabolic PDE.
76. Distinguish predictor and corrector formulae in solving an ordinary differential equation.
77. Discretise the following equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

using the finite difference method.

78. Discussed Runge-Kutta 4th order method to solve a pair of first-orderordinary differential equations.
79. Explain a finite difference method to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad t > 0, \qquad 0 < x < 1$$

where initial conditions $u(x, 0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{(x,0)} = g(x),\ 0 < x < 1$ and boundary conditions $u(0, t) = \varphi(t)$ and $u(1, t) = \psi(t),\ t \geq 0$.

80. Solve the following system of equations $x = (8x - 4x^2 + y^2 + 1)/\ 8$ and $y = (2x - x^2 + 4y - y^2 + 3)/\ 4$ starting with $(x_0, y_0) = (1.1, 2.0)$, using Seidel iteration method.
81. Given $\frac{dy}{dx}$ =x – y$^2$ with x=0, y=1. Find y(0.2) by second and fourth order Runge-Kutta methods.
82. Discussed Runge-Kutta 4th order method to solve a second-order initial value problem.
83. Describe the finite difference method to solve the following BVP $p(x)\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} + r(x)y = s(x), x_0 \leq x \leq x_n,\ \ y(x_0) = \alpha,\ y(x_n) = \beta.$

84. Solve the heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, subject to the boundary conditions u(0, t)=0, u(1, t)=2t, and initial condition u(x, 0)= $\frac{x}{2}$.

85. Solve the following differential equation

$$\frac{dy}{dx} = x + y,\ y(0) = 1$$

Find the value of y(0.1) by Picard method.

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